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Magnetic Moment Fields in Dense Relativistic Plasma Interacting with Laser Radiations

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Abstract

Theory of the generation of magnetic moment field from resonant interaction of three high frequency electromagnetic waves in un-magnetized dense electron plasma is developed including the relativistic change of electron mass. It is shown that the inclusion of relativistic effect enhances the magnetic moment field. For high intensity laser beams this moment field may be of the order of a few mega gauss. Such a high magnetic field can considerably affect the transport of electrons in fusion plasma.

Keywords: Inverse Faraday effect; Resonant excitation; Relativistic effect.

1. Introduction

In laser produced plasmas self-generated magnetic fields [1-5] have been of much interest because of their role in the design of inertial confinement fusion (ICF) targets. This magnetic field can affect strongly several transport mechanisms which influence the performance of ICF target [2]. For example the magnetic moment field enhances the anomalous diffusion of plasma in presence of electromagnetic (EM) waves. To explain the origin of this magnetic moment field a variety of mechanisms have been proposed [2]. One possible mechanism of magnetic field generation in laser produced plasma is the inverse Faraday effect (IFE) mechanism.

When a circularly polarized high intensity electromagnetic (EM) wave passes through plasma the wave field forces the plasma particles to gyrate in orbits that depend on the wave intensity, frequency and the plasma density. This gyration of charged particles produces solenoidal current and hence induces a magnetic field which, depending on the state of polarization, is either parallel or antiparallel to the direction of propagation of the wave. The generation of this magnetic field is referred to as the inverse Faraday effect. Originally IFE was conceived as the generation of magnetic field from the magnetic moments induced only by the electric current of circularly polarized waves in crystals and plasmas. However the effect is more general .This magnetic moment field may be generated by any types of bending of plasma constituents by waves. The magnetic moment field was first found in the interaction of a circularly polarized wave with crystals by Pershan [6] , Pershan et al [7] and others while that with plasmas was found by Deschamps et al [8], Chian[9] and others. Steiger and Woods [10] studied the effect for a circularly polarized high intensity laser radiation interacting with a dense electron plasma. Recently Ali et al [11] have shown that IFE can also occur for linearly polarized radiation. They have calculated the quasi-static axial magnetic field generated by a laser propagating in plasma by considering both the spin and orbital angular momentum of the laser pulse. This effect is more pronounced for circularly polarized light than for any other state of polarization. Resonant excitation of magnetic moment field (REMF) was considered by Chakraborty et al [12]. They calculated the magnetic moment field generated through nonlinear resonant interaction of three high frequency EM waves in a collisionless unmagnetized electron plasma considering electron orbital motion to be non-relativistic. But in the optical frequency range and for laser beam of high intensities the electron orbital speed becomes extremely high and relativistic variation of mass of the electrons becomes important. In this case the induced magnetic moment field becomes so large that its effect on the orbital motion of plasma particles cannot be neglected. In fact it then effectively controls the wave induced features in plasma. According to Steiger and Woods [10] the enhancement of plasma transparency i.e. the increase in critical plasma density with radiation intensity is predominantly caused by the relativistic change of electron mass. High current laser powers are able to accelerate particles to highly relativistic speeds. Furthermore, pulse self compression in laser plasma systems may play an important

role in attaining power levels well above laser current limits [13]. So it remains important to consider the theory of resonant excitation of magnetic moment field including the effect of the relativistic variation of electron mass.

The motivation of the present paper is to consider the theory of generation of magnetic moment field from resonant interaction of three high frequency transverse EM waves in an unmagnetized dense electron-ion plasma including the relativistic change of electron mass. The induced magnetic moment field is proportional to the vector product of wave-induced displacement and current. Due to prescribed frequency matching of the three waves the generated moment field has a zero-th harmonic component. It is shown that the inclusion of relativistic change of electron mass enhances the generated resonant magnetic moment field. This moment field, from wave induced bending of direction of motion of constituents of plasma, is important where it grows to large values with time. For high intensity laser beams interacting with a dense plasma this component may have a value of the order of a few mega gauss. The magnetic moment field in such a situation effectively controls the wave induced features of the plasma. It affects the transport process in plasma and promises the development of an ultrafast laser-controlled magnetic writing process.

2. Formulation

We consider an unmagnetized, collisionless and relativistic plasma interacting with three transverse intense electromagnetic waves all propagating along the positive z-axis. The primary coupling between the high frequency waves and the plasma is given by the interaction between the electrons and the electric field of the waves. We may therefore neglect to a first approximation the wave's fluctuating magnetic field. Compared to the large velocity acquired by the electrons the ion velocity remains well below electron velocity. The ion motion will then have little effect on wave propagation. For this we assume the ions to form a uniform stationary background having density N_0 . For simplicity we also neglect any quantum mechanical effects. Under such assumptions the basic equations governing the system dynamics are the following:

$$\frac{\partial N}{\partial t}+\vec{\nabla}.(N\vec{u})=0 \tag{1}$$

$$\frac{\partial \vec{u}}{\partial t}+\frac{e}{m}\vec{E}+\frac{C_s^2}{N_0}\vec{\nabla}N=-(\vec{u}.\vec{\nabla})\vec{u}-\frac{e}{mc}(\vec{u}\times\vec{B}) \tag{2}$$

$$\vec{\nabla}\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \tag{3}$$

$$\vec{\nabla}\times\vec{B}=\frac{1}{c}\frac{\partial\vec{E}}{\partial t}-\frac{4\pi}{c}.eN\vec{u} \tag{4}$$

$$\vec{\nabla}.\vec{E}=-4\pi e(N-N_0) \tag{5}$$

where N is the electron number density in the perturbed state, N_0 is the number density in the unperturbed state, $\vec{u}$ is the electron fluid velocity, C_s is the electron acoustic speed, $\vec{E}$ and $\vec{B}$ are respectively the self-consistent electric and magnetic fields in the plasma; $m=m_0(1-u^2/c^2)^{-1/2}, m_0$ being the rest mass of the electron and the other symbols have their usual meanings.

We assume that all the three transverse EM waves are left elliptically polarized and are propagating along the positive z-axis. So the wave electric field can be represented as

$$\begin{aligned}\vec{E_1}=(&a_1\cos\theta_1+a_2\cos\theta_2+a_3\cos\theta_3,\\ &b_1\sin\theta_1+b_2\sin\theta_2+b_3\sin\theta_3,0)\end{aligned} \tag{7}$$

where $\theta_j=k_jz-\omega_jt(j=1,2,3);k_j$ and ω_j are respectively the wavenumber and frequency of the j-th transverse elliptically polarized wave; a_j and b_j represent the laser amplitudes.

We assume the following frequency matching condition,

$$\omega_1+\omega_2=\omega_3 \tag{8}$$

Each of the gyrating electrons contributes a magnetic dipole moment [14]

$$\vec{\mu}=\frac{1}{2c}(\vec{r}\times\vec{j}) \tag{9}$$

where $\vec{r}$ is the position vector of the electron and $\vec{j}$ is its current. The magnetic induction generated by the gyrating electrons is then given by

$$\vec{B_{in}}=4\pi N\vec{\mu}=\frac{4\pi N}{2c}(\vec{r}\times\vec{j}) \tag{10}$$

The non-oscillating part of $\vec{B_{in}}$ is obtained as

$$\langle\vec{B_{in}}\rangle=-\frac{4\pi Ne}{2c}\langle(\vec{\xi}\times\vec{u})\rangle \tag{11}$$

where $\vec{\xi}$ is the wave induced displacement.

We now make the following perturbation expansion for the field quantities:

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$$\begin{aligned} N &= N_0 + \varepsilon N_1 + \varepsilon^2 N_2 + ... \\ \vec{u} &= \vec{u}_0 + \varepsilon \vec{u_1} + \varepsilon^2 \vec{u_2} + ... \\ \vec{E} &= 0 + \varepsilon \vec{E_1} + \varepsilon^2 \vec{E_2} + ... \\ \vec{B} &= 0 + \varepsilon \vec{B_1} + \varepsilon^2 \vec{B_2} + ... \\ \vec{\xi} &= 0 + \varepsilon \vec{\xi_1} + \varepsilon^2 \vec{\xi_2} + ... \end{aligned} \quad (12)$$

where the subscripts 0, 1, 2, etc. corresponds to the equilibrium, first order, second order, etc values of the respective quantities.

Using (7) and (12) in Eqs. (1)- (5) and solving up to first order for the field variables we get

$$\vec{B_1} = \frac{m_0 c^2}{e}[-\sum_j \beta_j k_j \sin\theta_j, \sum_j \alpha_j k_j \cos\theta_j, 0] \quad (13)$$

$$\vec{u_1} = \frac{c}{\gamma_0}[\sum_j \alpha_j \sin\theta_j, -\sum_j \beta_j \cos\theta_j, 0] \quad (14)$$

$$\vec{\xi_1} = \frac{c}{\gamma_0}[\sum_j \frac{\alpha_j}{\omega_j}\cos\theta_j, \sum_j \frac{\beta_j}{\omega_j}\sin\theta_j, 0] \quad (15)$$

$$N_1 = 0 \quad (16)$$

where

$$(\alpha_j, \beta_j) = \frac{e}{m_0 c \omega_j}(a_j, b_j) \quad (17\text{ a})$$

and

$$\gamma_0 = 1/(1 - u_0^2/c^2)^{1/2} \quad (17\text{b})$$

in which u_0 is the drift velocity of the plasma.

Now assuming the frequency matching condition (8) and solving Eqs. (1)- (5), correct up to second order of small quantities we find that

$$\begin{aligned} \vec{u_2} = [0,\, 0,\, & \delta_1^+(\omega_1+\omega_2)\cos(\theta_1+\theta_2) \\ &+ \delta_2^+(\omega_2+\omega_3)\cos(\theta_2+\theta_3) \\ &+ \delta_3^+(\omega_3+\omega_1)\cos(\theta_3+\theta_1) + \delta_1^-(\omega_1-\omega_2)\cos(\theta_1-\theta_2) \\ &+ \delta_2^-(\omega_2-\omega_3)\cos(\theta_2-\theta_3) + \delta_3^-(\omega_3-\omega_1)\cos(\theta_3-\theta_1)] \end{aligned} \quad (18)$$

$$\begin{aligned} \vec{\xi_2} = [0, 0, & \delta_1^+\cos(\theta_1+\theta_2) + \delta_2^+\cos(\theta_2+\theta_3) \\ &+ \delta_3^+ c\text{os}(\theta_3+\theta_1) + \delta_1^-\cos(\theta_1-\theta_2) \\ &+ \delta_2^-\cos(\theta_2-\theta_3) + \delta_3^-\cos(\theta_3-\theta_1)] \end{aligned} \quad (19)$$

where

$$\delta_1^{\pm} = \frac{(\omega_1 \pm \omega_3)\{(\omega_1+\omega_3) - (k_1+k_3)u_{0z}\}(\alpha_1^2 \mp \beta_1^2)}{\gamma_{0z}^2(k_1 \pm k_3)[4\{\omega_1+\omega_3) - (k_1+k_3)u_{0z}\}^2 + \omega_p^2/\gamma_{0z}^2]}$$

$$\delta_2^{\pm} = \frac{(\omega_2 \pm \omega_3)\{(\omega_2+\omega_3) - (k_2+k_3)u_{0z}\}(\alpha_2^2 \mp \beta_2^2)}{\gamma_{0z}^2(k_2 \pm k_3)[4\{\omega_2+\omega_3) - (k_2+k_3)u_{0z}\}^2 + \omega_p^2/\gamma_{0z}^2]}$$

$$\delta_3^{\pm} = \frac{(\omega_3 \pm \omega_1)\{(\omega_3+\omega_1) - (k_3+k_1)u_{0z}\}(\alpha_3^2 \mp \beta_3^2)}{\gamma_{0z}^2(k_3 \pm k_1)[4\{\omega_3+\omega_1) - (k_3+k_1)u_{0z}\}^2 + \omega_p^2/\gamma_{0z}^2]} \quad (20)$$

Using (9) the magnetic moment field generated due to nonlinear interaction of three transverse waves in relativistic plasma are obtained as

$$\langle B_{ind}\rangle_x = -\frac{4\pi e N_0}{2c\gamma_0^3}[\xi_{1y}u_{2z} - \xi_{1z}u_{2y} + \xi_{2y}u_{1z} - \xi_{2z}u_{1y}]$$

$$\langle B_{ind}\rangle_y = -\frac{4\pi e N_0}{2c\gamma_0^3}[\xi_{1z}u_{2x} - \xi_{1x}u_{2z} + \xi_{2z}u_{1x} - \xi_{2x}u_{1z}]$$

$$\langle B_{ind}\rangle_z = -\frac{4\pi e N_0}{2c\gamma_0^3}[\xi_{1x}u_{2y} - \xi_{1y}u_{2x} + \xi_{2x}u_{1y} - \xi_{2y}u_{1x}] \quad (21)$$

Now using equations (14),(15),(18) and(19) in equation(21) we get the magnetic moment fields correct up to second order as

$$\langle B_{ind}\rangle_x = \frac{\pi e N_0 c^2}{\gamma_0}[\frac{\beta_1 B_{23}^-}{(\omega_2+\omega_3)^2} + \frac{\beta_2 B_{13}^-}{(\omega_3+\omega_1)^2} + \frac{\beta_3 B_{12}^+}{(\omega_1+\omega_2)^2}]\sin(\Delta k z) \quad (22)$$

$$\langle B_{ind}\rangle_y = \frac{\pi e N_0 c^2}{\gamma_0}[\frac{\alpha_1 B_{23}^-}{(\omega_2+\omega_3)^2} + \frac{\alpha_2 B_{13}^-}{(\omega_3+\omega_1)^2} + \frac{\alpha_3 B_{12}^+}{(\omega_1+\omega_2)^2}]\sin(\Delta k z) \quad (23)$$

$$\langle B_{ind}\rangle_z = 0 \quad (24)$$

where

$$B_{ij}^{\pm} = \frac{(\alpha_i\alpha_j \pm \beta_i\beta_j)(k_i \pm k_j)}{c^2(k_i \mp k_j)^2 - (\omega_i \pm \omega_j)^2 + \omega_p^2} \quad (25)$$

i, j=1, 2, 3 and $i \neq j$

The symbol $\langle B_{ind}\rangle$ stands for the average of the quantity B_{ind} over a time period. Thus $\langle B_{ind}\rangle$ is the non-oscillating zero-th harmonic part of the induced moment field. The quantity Δk in equations (22) and (23) is given as

$$\Delta k = k_1 + k_2 - k_3 = [(\omega_1^2 - \omega_p^2)^{1/2} + (\omega_2^2 - \omega_p^2)^{1/2} - (\omega_3^2 - \omega_p^2)^{1/2}] \quad (26)$$

Or,

$$\Delta k \approx -\frac{\omega_p^2}{2c}(\frac{1}{\omega_1} + \frac{1}{\omega_2} - \frac{1}{\omega_3}) \quad (27)$$

3. Results and Discussions

The expressions (22)-(24) give the magnetic moment field generated due to nonlinear resonant interaction of three transverse EM waves with a dense electron-ion plasma

including relativistic effect. It is obvious that moment field exists only in the lateral direction with respect to the common direction of propagation of the three transverse waves. However if one of the interacting waves is longitudinal in nature ,for example which may be a high frequency electron acoustic wave, then the component of the magnetic moment field along the direction of propagation of the wave will be non-zero. The induced static magnetic field enhances anomalous diffusion of plasma in the presence of wave fields. The induced magnetic field with relativistic effect is higher than that obtained in the non-relativistic case. The drift velocity of electrons either parallel or antiparallel to the direction of wave propagation has significant effect on the magnitude of the generated magnetic field. This is due to the fact that the drift velocity modifies the total force driving the electrons in its orbit and it contributes directly to the relativistic mass increase. Considering a neodymium –glass laser having intensity $I = 1.7\times10^{15} W/cm^2$ and wavelength $\lambda = 1.06\mu m$ the induced magnetic field for a typical plasma with number density $N_0 = 10^{21}/cm^3$ is calculated for different values of the streaming factor u_0 /c .The result is shown graphically in Fig.1. Obviously the induced magnetic field is of the order of mega gauss and it increases with the increase in the relativistic streaming factor. However it should be kept in mind that the theoretical model considered in this paper is applicable only for weakly relativistic case. For strongly relativistic case we require to develop new theoretical model. Comparing with the results with the nonrelativistic case [7] it is found that the inclusion of relativistic effect with u_0 /c=0.4 gives rise to a magnetic field which is about 15% higher than that in the nonrelativistic case. The mismatch between the wavenumbers causes a sinusoidal spatial variation of magnetization along the direction of propagation of the waves which is perpendicular to the direction of magnetization. The distance between successive regions of maximum and minimum of magnetization is of the order of c/ω. As a consequence of it there are charge dependent drift and related current flow.

The region where the field generation occurs is small compared to the largest of the wavelength of the involved waves. Such common region might exist where the waves cross each other or where these are reflected [12].Since we have considered the interaction of plasma with transverse modes only the pressure term in Eq.(2) does not contribute to the induced magnetic moment field. However if one of the interacting waves is longitudinal in nature then this term would definitely give a significant contribution. The resonant excitation of magnetic moment field is an almost instantaneous process at the time of switch -on of the wave fields in the region where the frequency matching occurs. For this the present analysis will describe the process occurring only over a short duration of time. The analysis made in this paper for a model plasma can to a good approximation be applied to the conducting electrons of a metallic material, which can be considered as a collisionless plasma at least on the time scale given by the period of the high frequency field.

Finally we would like to point out that the study of IFE is important not only for understanding the fundamental processes involved in magnetization dynamics but also for the fact that such studies may pave the way towards an ultrafast focused laser-controlled magnetic writing process that could eventually replace the today's clumsy data storage system using microscopic coils and write heads that may generate undesired fringing fields.

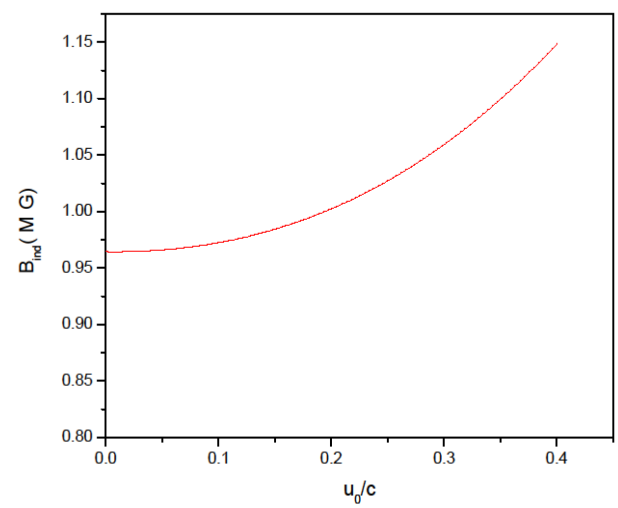

Figure 1: Variation of induced magnetic moment field with the relativistic streaming factor u_0/c

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